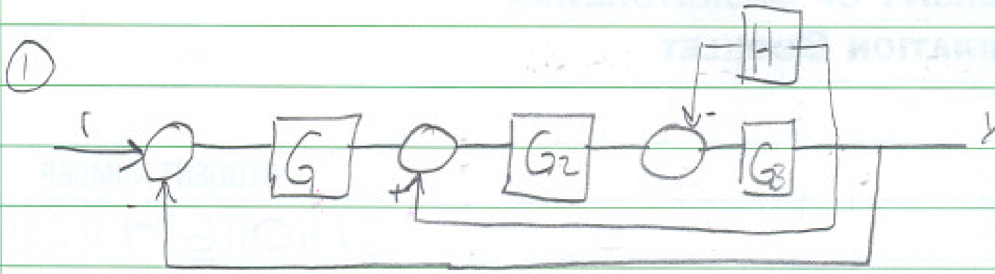
①



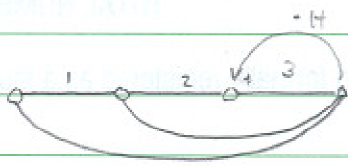
$P = G_1 G_2 G_3$

$L_1 = -G_3 H$

$L_2 = G_2 G_3$

$L_3 = -G_1 G_2 G_3$

$$\frac{Y}{R} = \frac{P}{1 + G_3 H + G_1 G_2 G_3 - G_2 G_3}$$

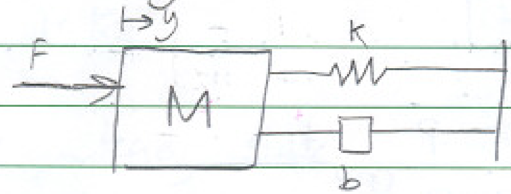


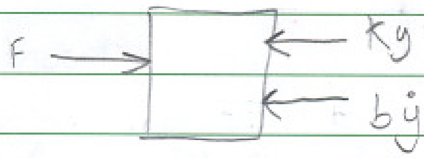
Transfer function:

$$\frac{Y}{R} = \frac{G_1 G_2 G_3}{1 + G_3 H + G_1 G_2 G_3 - G_2 G_3}$$

6.0

(2) Looking at mass





$M = 2.4\ kg$
$k = 30\ N/m$
$b = 10\ Ns/m$
$F = 1N$

$$F - ky - b\dot{y} = M\frac{d^2y}{dt}$$

$\delta = 2\% = 0.02$

$$F - kY - bsY = Ms^2Y$$

$$F = Y(Ms^2 + bs + k)$$

$$\boxed{\frac{Y(s)}{F} = \frac{1}{Ms^2 + bs + k}}$$

general form $\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

5.5

$$\frac{Y}{F} = \frac{1/m}{s^2 + \frac{b}{m}s + k/M}$$

Steady state value $= \dfrac{1}{km} =$ $\cancel{72}$ $\dfrac{kgN}{m}$

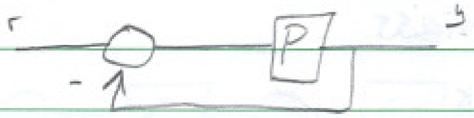
$$2\zeta\omega_n = b/m = \zeta\omega_n = \frac{b}{2M} \qquad \omega_n = \sqrt{k}$$

or $\omega_n = \sqrt{1/m}$

T_s = settling time $= \dfrac{4}{\zeta\omega_n}$ for $\delta = 2\%$

$$T_s = \frac{4(2M)}{b} \Rightarrow \boxed{T_s = 1.92 \text{ seconds}}$$

(3)



Transfer Function $= \dfrac{P}{1+P}$

to be stable:

✓ - no zero-pole cancellations

- no pole roots in the right hand plane

(a) $P(s) = \dfrac{161}{(s+5)(s+4)}$

$T(s) = \dfrac{161}{(s+5)(s+4)+161}$

$den = s^2 + 9s + (20 + 161) = s^2 + 9s + 181$

$roots = \dfrac{-9 \pm \sqrt{81 - 4(181)}}{2}$

$roots = -4.5 \pm \sqrt{-160.75}$

all roots in the left hand side of s-plane => STABLE

Unit Step Input

$k_p = \lim_{s\to 0} P(s) = \dfrac{161}{20} = \underline{8.05}$

$$e_{ss} = \frac{1}{1+K_p} = 0.1105$$

Error at steady $\text{state}_{u(t)}$ = 11.05 %

Ramp Input.

$$K_v = \lim_{s\to 0} s P(s) = \lim_{s\to 0} \frac{s\, 161}{(s+5)(s+4)} = 0$$

P(s) is type 0, so this is right.

The error signal $= \frac{1}{K_v} = \infty$.

3.0

$s^2+9s+181$

$$\begin{array}{c|cc} s^2 & 1 & 181 \\ s & 9 & 0 \\ 1 & 181 & \end{array} \qquad \frac{0-181(9)}{-9} = 181$$

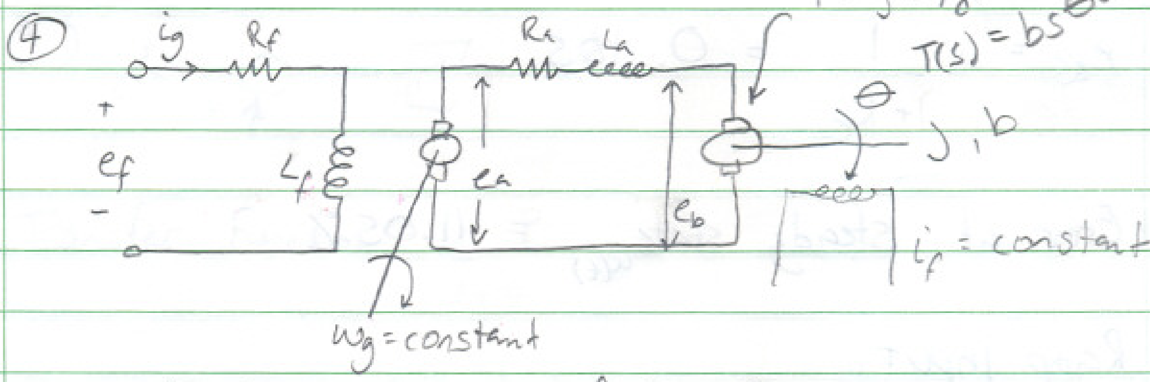
no sign changes $\Rightarrow$ STABLE

part b?

$T = j\,d\omega/dt + b\omega$

$T(s) = bs\Theta(s) + js^2\Theta(s)$

(4)



$e_a = K_g i_g$ $\qquad$ find $\dfrac{\Theta}{e_f}$

$P_{elec} = e_a i_a$

$P_{mech} = T\omega_g$

$e_a i_a = T\omega_g$

$T = \dfrac{e_a i_a}{\omega_g} = \dfrac{K_g i_g i_a}{\omega_g}$ $\qquad K_1 = \dfrac{1}{\omega_g}$

$T = K_1 K_g i_g i_a$

$e_a - e_b = i_a R_a + L\dfrac{di_a}{dt}$

$E_a - E_b = I_a R_a + Ls I_a$

$E_a - E_b = I_a(R_a + Ls)$

$I_a = \dfrac{E_a - E_b}{(R_a + Ls)}$

$E_a = K_g I_g$

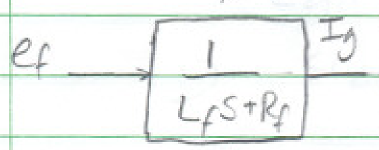
$I_a = \dfrac{K_g I_g - E_b}{R_a + Ls}$

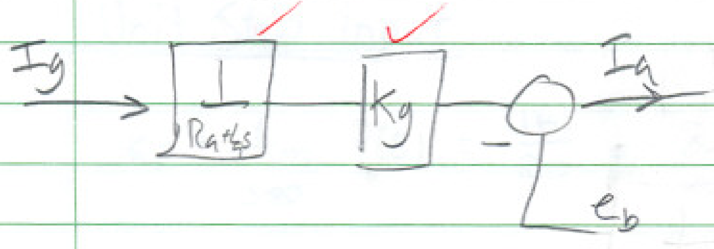
$e_f = i_g R_f + L_f\dfrac{di_g}{dt}$

$e_f(s) = I_g(s)R_f + L_f s\, I_g(s)$

$E_f(s) = I_g(s)(L_f s + R_f)$

$I_g = \dfrac{E_f(s)}{L_f s + R_f}$

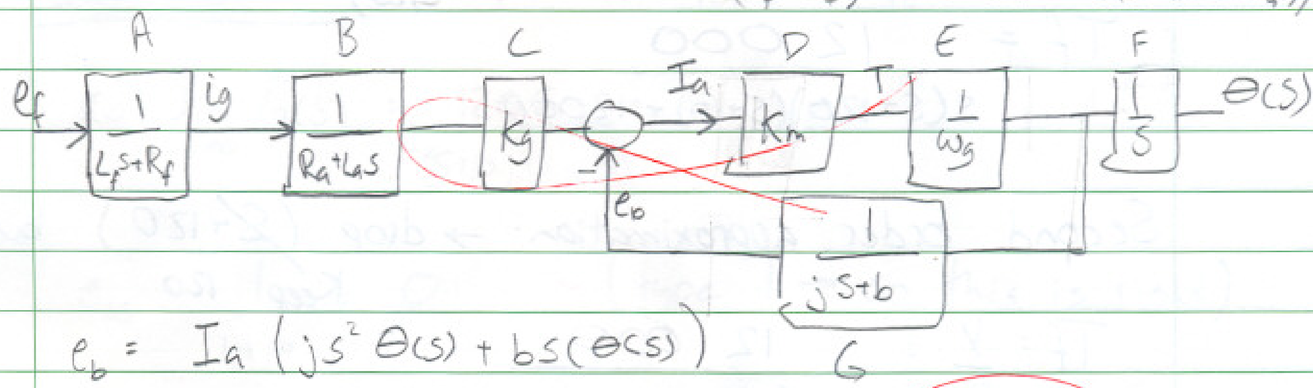




$i_a = K_2 K i_f = K_m$

$$\Theta(s) = \frac{T(s)}{(js^2 + bs)} = \frac{T(s)}{s(js+b)}$$

$$T(s) = K_1 K_g I_g I_a = K_1 K_g \frac{E_f}{(L_f s + R_f)} \left(\frac{-E_b}{R_a + L_a s} + \frac{K_g E_f}{(L_f s + R_f)(R_a + L_a s)} \right)$$



$e_b = I_a \left(js^2 \Theta(s) + bs(\Theta(s)) \right)$

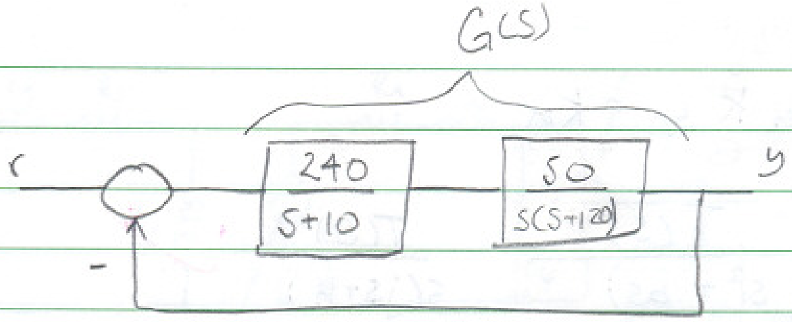
3.5

$T = K K_f i_f i_a = K_m i_a$

$$\frac{\Theta(s)}{E_f(s)} = \frac{ABCDEF}{1 + DEG} = \frac{ABCFD}{\frac{1}{EG} + D}$$

$$\cancel{\frac{\Theta(s)}{E_f(s)} = \frac{K_g K_m}{s(L_f s + R_f)(R_a + L_a s)\omega_g[(\omega_g)(js+b) + K_m]}}$$

(5)



Real Transfer function $= \dfrac{G(S)}{1+G(S)}$

$$Tf = \frac{12\,000}{S(S+120)(S+10)+12000}$$

Second order approximation: $\rightarrow$ drop $(\cancel{S}+120)$ but keep 120

$$Tf = \frac{Y}{R} = \frac{12\,000}{S(S+10)(120)+12000}$$

$$Y/R = \frac{12\,000}{120S^2+1200S+12000}$$

$$\boxed{Y/R = \frac{100}{S^2+10S+100}}$$

$2\zeta\omega_n = 10$ $\qquad\qquad$ $\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$\zeta\omega_n = 5$

$\zeta = 0.5$

$$PO = \exp\left(\frac{-0.5\pi}{(1-0.5^2)^{1/2}}\right) \Rightarrow PO = 0.1630$$

$\boxed{\text{Percent Overshoot} = 16.30\%}$

$$T_S = \frac{4}{\zeta \omega_n} \quad (\delta = 2\%) \Rightarrow \boxed{T_S = 0.8 \text{ seconds}}$$

$$T_p = \frac{\pi}{\omega_n (1-\zeta^2)^{1/2}} \Rightarrow \boxed{T_p = 0.3628 \text{ seconds}}$$

Step Input: (all roots in LHP $\rightarrow -5 \pm 8.66j$)

$$k_p = \lim_{s \to 0} G(s) = \frac{12000}{(s+10)s(120)} = \infty$$

$$e_{ss} = \frac{1}{1+\infty} = 0$$ ✓ (type 1 → so this is right)

6.0

ramp input

$$k_v = \lim_{s \to 0} s\,G(s) = \frac{\cancel{s}\,12000}{(\cancel{s}+10)\cancel{s}(120)} = \frac{12000}{1200} = 10$$ ✓

$$e_{ss} = \frac{1}{k_v} = 0.1 = \underline{\underline{10\%}}$$ ✓